

**ERGODIC THEORY AND ITS APPLICATIONS**  
**18–22 JULY 2016, SYDNEY**  
**ABSTRACTS OF TALKS**

**Monday, July 18**

**9:30 Anthony Dooley**

**Title:** Classification of non-singular systems and critical dimension

**Abstract:** "In work with Hamachi, I have shown that every non-singular measurable dynamical system is orbit equivalent to a Markov odometer on a Bratteli-Vershik system which is uniquely ergodic, minimal and induced as a subsystem of a full odometer. At the same time, Mortiss and I introduced a new invariant, the critical dimension, for metric isomorphism of non-singular systems: it is given by the rate of growth of sums of Radon-Nikodym derivatives. The critical dimension induces a natural form of equivalence between systems, which we call Hurewicz equivalence. It turns out that the critical dimension is an invariant for Hurewicz equivalence of Markov odometers of finite width, and this leads to a classification theorem. It is also related to Hausdorff dimension of the measure. In recent work, Kieran Jarrett and I are investigating this invariant for actions of amenable groups."

**11:00 Andy Hammelindl**

**Title:** Advances in Stable Ergodicity

**Abstract:** "Anosov systems are stable under perturbation and all volume-preserving Anosov systems are ergodic. These systems are therefore examples of stably ergodic dynamical systems; that is, systems which remain ergodic under small perturbations. For decades, these were the only known examples. Then, in the 1990s new stably ergodic examples arose which are only partially hyperbolic. In this talk, I will overview the links between stable ergodicity and partially hyperbolic and discuss my own work in the area."

**14:00 James Parkinson**

**Title:** "A Multiplicative Ergodic Theorem for p-adic Lie groups "

**Abstract:** "The celebrated Multiplicative Ergodic Theorem (MET) of Oseledets shows that under a finite first moment assumption the product of random real iid matrices behaves asymptotically like the sequence of powers of some fixed positive definite symmetric matrix. Kaimanovich observed that this property can be expressed in purely geometric terms using the symmetric space associated to  $GL_n(\mathbb{R})$ . This led to the notion of a 'regular sequence' in a symmetric space, and by characterising these sequences in terms of spherical and horospherical coordinates Kaimanovich obtained a MET for noncompact semisimple real Lie groups with finite centre, generalising the original theorem of Oseledets.

In this talk we will discuss a p-adic analogue of this story. In this setting the symmetric space is replaced by the affine building of the p-adic group. We define regular sequences in affine buildings, and give a characterisation of these sequences in terms of analogues of the spherical and horospherical coordinates from the real theory. We then discuss applications to a MET for Lie groups defined over p-adic fields. This is joint with W. Woess."

**15:30 Lawrence Reeves**

**Title:** Commutators in groups of piecewise projective homeomorphisms

**Abstract:** The Lodha-Moore groups are finitely presented counterexamples to the von Neumann-Day conjecture. They appear as subgroups of a group of piecewise projective maps constructed by Monod. We study their commutators and second commutators, showing some of them are simple. This is joint work with Jose Burillo and Yash Lodha.

**Tuesday, July 20**

**9:30 Faustin Adiceam**

**Title:** On the minimum of a positive definite quadratic form over non-zero lattice points. Theory and applications.

**Abstract:** "Let  $\Sigma_d^{++}(\mathbb{R})$  be the set of positive definite matrices with determinant 1 in dimension  $d \geq 2$ . Identifying two  $SL_d(\mathbb{Z})$ -congruent elements in  $\Sigma_d^{++}(\mathbb{R})$  gives rise to the space of reduced quadratic forms of determinant one, which in turn can be identified with the locally symmetric space  $X_d := SL_d(\mathbb{Z}) \backslash SL_d(\mathbb{R}) / SO_d(\mathbb{R})$ . Equip the latter space with its natural probability measure coming from the Haar measure on  $SL_d(\mathbb{R})$ . In 1998, Kleinbock and Margulis established very sharp estimates for the probability that an element of  $X_d$  takes a value less than a given real number  $\delta > 0$  over the non-zero lattice points  $\mathbb{Z}^d \setminus \{\mathbf{0}\}$ . This talk will be concerned with extensions of such estimates to a large class of probability measures arising either from the spectral or the Cholesky decomposition of an element of  $\Sigma_d^{++}(\mathbb{R})$ . The sharpness of the bounds thus obtained are also established for a subclass of these measures. This theory has been developed with a view towards application to Information Theory. Time permitting, we will briefly introduce this topic and show how the estimates previously obtained play a crucial role in the analysis of the performance of communication networks.

This is work joint with Evgeniy Zorin (University of York)."

**11:00 David Simmons**

**Title:** Extremality and dynamically defined measures

**Abstract:** We consider a class of measures from Diophantine approximation known as *extremal* measures. The class of measures known to be extremal has expanded in recent years to include not only the Lebesgue measures of nondegenerate manifolds, but also various measures defined using conformal dynamics. In this talk I will describe this history as well as describing a new geometric condition which implies extremality, giving examples of dynamical measures satisfying this condition which could not previously be proven to be extremal. This work is joint with Tushar Das, Lior Fishman, and Mariusz Urbanski.

**14:00 Alina Ostafe**

**Title:** Disorderly behaviour of polynomial iterates

**Abstract:** "The underlying motif of the talk is showing various instances of the following principle: Polynomials have no respect for Law and Order.

More precisely, given a polynomial  $f$  over a field  $K$  and a structural set  $S \subset K$  defined in terms unrelated to  $f$ , it is natural to expect that the orbits of  $f$  have a finite intersection with  $S$ . For example, if  $S$  is an orbit of another polynomial this is known as a problem about orbit intersections, which has recently been studied by Ghioca, Tucker and Zieve. One can also consider the multivariate generalisation of this question. For example, if the set  $S$  is an algebraic variety, this falls within the so-called dynamical Mordell-Lang conjecture.

We are interested in finiteness results or, failing this, in bounding the frequency of such intersections, both in the zero and positive characteristics, as well as for both univariate and multivariate cases. We shall also discuss several open questions in this direction."

**15:30 Nikos Frantzikinakis**

**Title:** Multiple correlation sequences, nilsequences, and arithmetic sequences

**Abstract:** "Multiple correlation sequences appear naturally when studying recurrence and convergence phenomena in ergodic theory and related problems in combinatorics and number theory. Several interesting problems rest upon a detailed understanding of their intrinsic structure. We shall see that under very general hypothesis any such correlation sequence is the sum of a nilsequence and an error term that is small in uniform density. We use this decomposition to prove convergence criteria for multiple ergodic averages which we then use to establish refinements of the multidimensional polynomial Szemerédi theorem and related mean convergence results. For instance, we show that in all these statements we can restrict the "common difference" to those integers that have an even (or an odd) number of distinct prime factors, and also give simple necessary and sufficient conditions for an arithmetic sequence to be a good universal weight for mean convergence. This is based on joint work with Bernard Host."

**Wednesday, July 20****9:30 Alexander Gorodnik****Title:** Distribution of rational points and ergodic theory.**Abstract:** "We discuss discrepancy of distribution of the set of rational points lying on algebraic groups, and describe an approach for obtaining upper and lower bounds on discrepancy. Our method is based on analysis of suitable ergodic averages and estimating spherical functions. This is a joint work A. Ghosh and A. Nevo."**11:00 Michael Björklund****Title:** Sturm und Rang**Abstract:** The talk will survey some recent joint work with A. Fish on the classification of "small" products of "large" sets in a general countable amenable group.

As an application of our results, we show that a sufficiently "spread-out" approximate subgroup of rank (Rang) 2 in any countable amenable group must be essentially "Sturmian" (explaining the pun in the title). All necessary terminology will be introduced.

Thursday, July 21

9:30 **Shahar Mozes**

**Title:** Divisibility properties of higher rank lattices

**Abstract:** In a joint work with Manfred Einsiedler we discuss a relationship between the dynamical properties of a maximal diagonalizable group  $A$  on certain arithmetic quotients and arithmetic properties of the lattice. In particular, we consider the semigroup of all integer quaternions under multiplication. For this semigroup we use measure rigidity theorems to prove that the set of elements that are not divisible by a given reduced quaternion is very small: We show that any quaternion that has a sufficiently divisible norm is also divisible by the given quaternion. Restricting to the quaternions that have norm equal to products of powers of primes from a given list (containing at least two) we show that the set of exceptions has subexponential growth.

11:00 **Riddhi Shah**

**Title:** Distal Actions on Locally Compact Groups

**Abstract:** We discuss some properties of distal actions on locally compact groups. We find a necessary and sufficient condition for an automorphism to be distal in terms of its contraction groups. We also relate distality and minimal orbit closure property. We classify pointwise distal groups in terms of behaviour of powers of probability measures on it, namely the concentration functions and shifted convolution property. We also relate such groups with those whose closed subgroups are unimodular.

12:00 **Colin Reid**

**Title:** Distal actions in totally disconnected, locally compact groups

**Abstract:** Let  $G$  be a group acting on a topological space  $X$ . A proximal pair is a pair  $(x,y)$  of distinct points in  $X$  such that the closure of the  $G$ -orbit of  $(x,y)$  intersects the diagonal of  $X \times X$ . A distal action is an action without proximal pairs (for example, an isometric action on a metric space). I will be talking about distal actions arising from conjugation inside totally disconnected, locally compact groups and their consequences. For example, using actions on coset spaces in  $G$ , one can give a sufficient condition on subgroups  $H$  of  $G$  for every ergodic continuous measure-preserving action of  $G$  on a Borel probability space to restrict to an ergodic action of  $H$ .

14:00 **CRE Raja**

**Title:** Periodic points of solenoidal automorphisms

**Abstract:** "Talk is based on a joint work with Sharan Gopal of BIT, Hyderabad. We give a characterization of the sets of periodic points of toral automorphisms. Then we describe the one-dimensional solenoids as the quotients of the (additive) group of adèles and characterise the sets of periodic points of automorphisms on these solenoids. It is also shown that an ergodic automorphism on a full solenoid has no periodic point other than the identity. "

15:30 **Kamil Bulinski**

**Title:** Spherical and Twisted multiple recurrence and configurations in positive density subsets of  $\mathbb{Z}^d$ .

**Abstract:** Magyar has shown that if  $B \subset \mathbb{Z}^d$  has positive upper density ( $d \geq 5$ ), then the set of squared distances  $\{\|b_1 - b_2\|^2 : b_1, b_2 \in B\}$  contains an infinitely long arithmetic progression. I will present various extensions of this. One involves showing the existence of other infinite families of Euclidean configurations (such as chains or, more generally, trees). The approach is Ergodic and in particular provides a new proof of Magyar's result, as well as some recent improvements by Lyall and Magyar. Secondly, I will present some results recently obtained with Bjorklund where we prove analogues of these aforementioned results for different quadratic forms and homogenous polynomials, which involves a "Twisted multiple recurrence" result that relies on deep results of Benoist-Quint on random walks on lattices of arithmetic groups.

**Friday, July 22**9:30 **Michael Cowling**

**Title:** On matrix coefficients of unitary representations of semisimple Lie groups

**Abstract:** "An action of a semisimple Lie group  $G$  on a measure space  $X$  gives rise to a unitary representation  $\pi$  of  $G$  on  $L^2(X)$ . It is of interest to know which irreducible unitary representations of  $G$  appear in the decomposition of  $\pi$ . It is sometimes relatively easy to find some information about the behaviour of the matrix coefficients  $\langle \pi(\cdot)\xi, \eta \rangle$ , where  $\xi$  and  $\eta$  lie in a dense subset of  $L^2(X)$ . The problem is then to translate this into information about the decomposition of  $\pi$ . This talk describes what I know about this problem."

11:00 **Jacqui Ramagge**

**Title:**

**Abstract:**