



THE UNIVERSITY OF SYDNEY
MATH2070 OPTIMISATION

Summer School

Quiz 1 – Week 1

2011

Name:

SID:

Time allowed: 30 minutes

Total Marks: 20

*Questions 1–2 are multiple choice questions, please circle the letter corresponding to your answer.
Questions 3–6 are extended answer questions. Please use pen and not pencil throughout the quiz paper.*

Multiple Choice Section

1. (1 mark) Consider the set of linear functions that are defined on a closed domain. Which of the following statements is true?
- (a) All extrema for linear functions are in the interior of their domain.
 - (b) All extrema for linear functions are on the boundary of their domain.
 - (c) A linear function always has a unique global extremum.
 - (d) A local extremum of a linear function is not necessarily a global extremum.
 - (e) none of the above.
2. (1 mark) Given the following tableau.

Z	x_1	x_2	x_3	x_4	x_5	RHS
1	1	0	2	0	1	6
0	-1	1	3	1	-2	3
0	1	2	1	0	2	6

Pick one of the original constraints from the list.

- (a) $x_1 + 2x_2 + x_3 \leq 6$
- (b) $3x_2 + 4x_3 \leq 9$
- (c) $x_1 + 2x_2 + x_3 \geq 6$
- (d) $3x_2 + 4x_3 \geq 9$
- (e) $x_1 + 2x_3 \leq 6$.

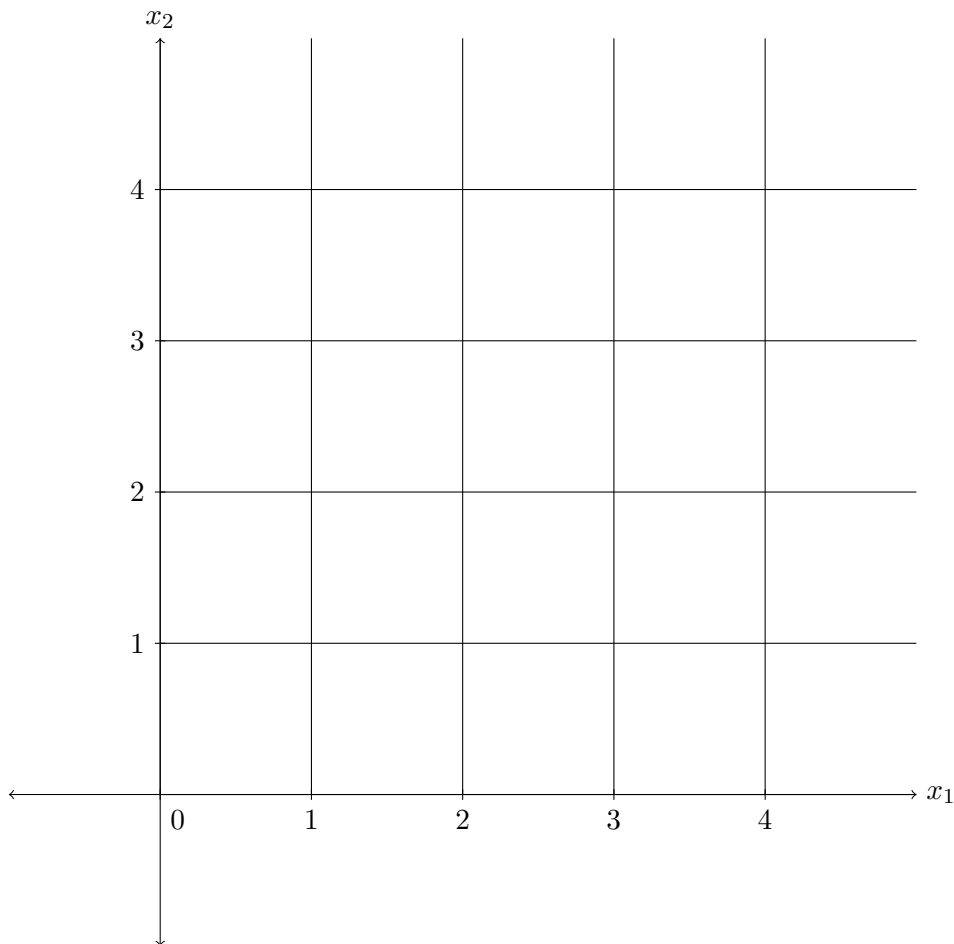
Extended Answer Section

Please write down the short answers to the questions as indicated. Please write any working on a separate page.

The next two questions refer to the Linear Programming problem.

$$\begin{aligned} \text{Max } Z &= x_1 + 2x_2 \\ \text{such that } x_1 &\leq 4 \\ -x_1 + 2x_2 &\leq 2 \\ x_1, x_2 &\geq 0. \end{aligned}$$

3. (5 marks) Solve the problem graphically. In the space below, draw the feasible region and identify all feasible corner points, along with the optimal solution vector (x_1^*, x_2^*) and the optimal level contour line of the objective function $Z(x_1^*, x_2^*) = Z^*$.



$$(x_1^*, x_2^*) =$$

$$Z^* =$$

4. (5 marks) Solve the previous problem using the simplex algorithm. Write the Initial tableau and the final tableau after completing all iterations of the simplex algorithm in the spaces provided below.

Initial Tableau (2 marks)

Z	x_1	x_2	x_3	x_4	RHS
1					
0					
0					

Final Tableau (3 marks)

Z	x_1	x_2	x_3	x_4	RHS
1					
0					
0					

5. (4 marks) Given the following final tableau

Z	x_1	x_2	x_3	x_4	x_5	RHS
1	1	0	0	2	1	42
0	1/2	0	1	1/3	-2	3
0	1	1	0	1	2	4

Answer the following questions in the boxes provided:

(a) What are the basic variables?

(b) What are the non-basic variables?

(c) What is the optimal solution vertex, $\mathbf{x}^* = (x_1^*, x_2^*, x_3^*)$?

(d) What is the optimal value, Z^* ?

6. (4 marks) Formulate the next paragraph into a linear programming problem with decision variables and constraints.

An actor, Bob, wants to gain weight to look the part for his role as an overweight truck driver in an upcoming movie. He decides to add servings of banana splits (S) and bacon rolls (B) to his diet. Each serving of S is 1,000 calories and each serving of B is 500 calories. Bob has a history of health problems and his doctor has recommended that he needs to control his cholesterol and sugar levels in his food intake. Each serving of S contains 1 unit of cholesterol and 3 units of sugar. Each serving of B contains 4 units of cholesterol and 1 units of sugar. His doctor recommends that his cholesterol be no larger than 10 units and his sugar levels should be at least 2 units and no larger than 9 units. What is the optimal choice of S and B to ensure Bob is ingesting as many calories as possible?

