Unit Outline: Math5310: Topics in Algebra Semester 2, 2024

Content: We will give an introduction to Algebraic Geometry, in terms of varieties. We focus primarily on the one-dimensional case of algebraic curves. The main objectives are deriving a local, embedding-free framework for algebraic geometry and the proving the Riemann-Roch theorem for curves. At the conclusion of the course the student should be well equipped to explore higher dimensional algebraic geometry and scheme-theoretic approaches.

We will follow closely the book 'Algebraic curves' by William Fulton, freely available at https://dept.math.lsa.umich.edu/~wfulton/CurveBook.pdf

Assumed Knowledge: Math3962/Math4062 and Math2022, basic notions of topological spaces and modules over rings.

Weekly Schedule:

Week 1: Algebraic sets and the Hilbert Basis Theorem.

Week 2: Nullstellensatz and polynomial maps.

Week 3: Rational functions, local rings and discrete valuation ring. Week 4: Multiple points and intersection numbers for plane curves.

Week 5: Projective varieties.

Week 6: Projective plane curves and Bézout's Theorem.
Week 7: Noether's Fundamental Theorem, Zariski topology.
Week 8: Morphisms of varieties and algebraic function fields.

Week 9: Rational maps and blowing up. Week 10: Resolutions of singularities.

Week 11: Riemann's Theorem.

Week 12: Riemann-Roch Theorem.

Week 13: Recap.